Project: 973
Name: Estimating Multi-Scale Instabilities in a Primitive Equation Model using Covariant Lyapunov Vectors
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The project aimed for computing linear instabilities using covarant Lyapunov vectors in the primitive equation model PUMA. Due to continuing problems in obtaining the tangent linear model of PUMA, we could not start the actual computations until October 2016. These problems existed because the employment of the automatic differentiation tool *FastOpt* for PUMA took much longer than anticipated. Due to these delays, it was not possible to perform studies of the CLVs themselves. Nevertheless, we could perform a basic investigation on the leading Lyapunov exponent (LLE) and the first two hundred Lyapunov exponents (LEs) in an aqua planet setup of PUMA. Our goal was to compare the convergence of the LLE for different resolutions to the Lorenz energy cycle of the atmosphere. For this, we varied the number of latitudes (nlat) and the number of vertical levels (nlev). Additionally, the pole-to-equator temperature gradient ΔT_{EP} was varied between 40 K and 80 K in 1 K steps. This wide range of ΔT_{EP} allowed for a complete picture from low to high degrees of chaos. Figure 1 shows the results for the leading Lyapunov exponent (LLE). Based on the all the results (nlat=32, 64,128 and nlev=10, 15, 20) which include also resolutions not shown here, we find that for a fixed horizontal resolution the LLE apparently converges for the same value of ΔT_{EP} when 10 or more vertical levels are chosen. We cannot conclude the same for the horizontal resolution, since here the LLE keeps growing for higher resolution. This means, even for resolutions usually used for investigating the general circulation in PUMA Boschi et al. (2014), there is no convergence of the fastest linear instability.

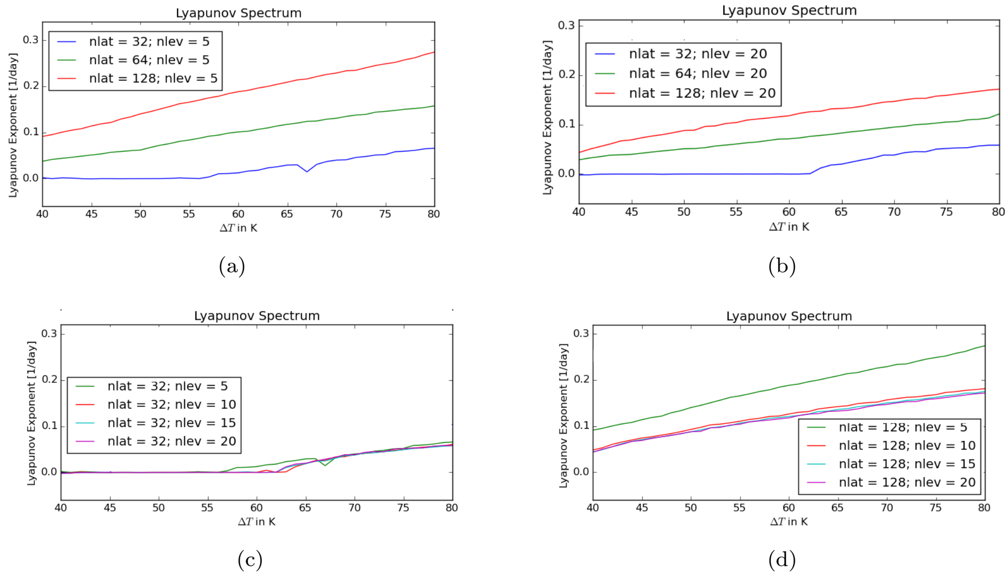
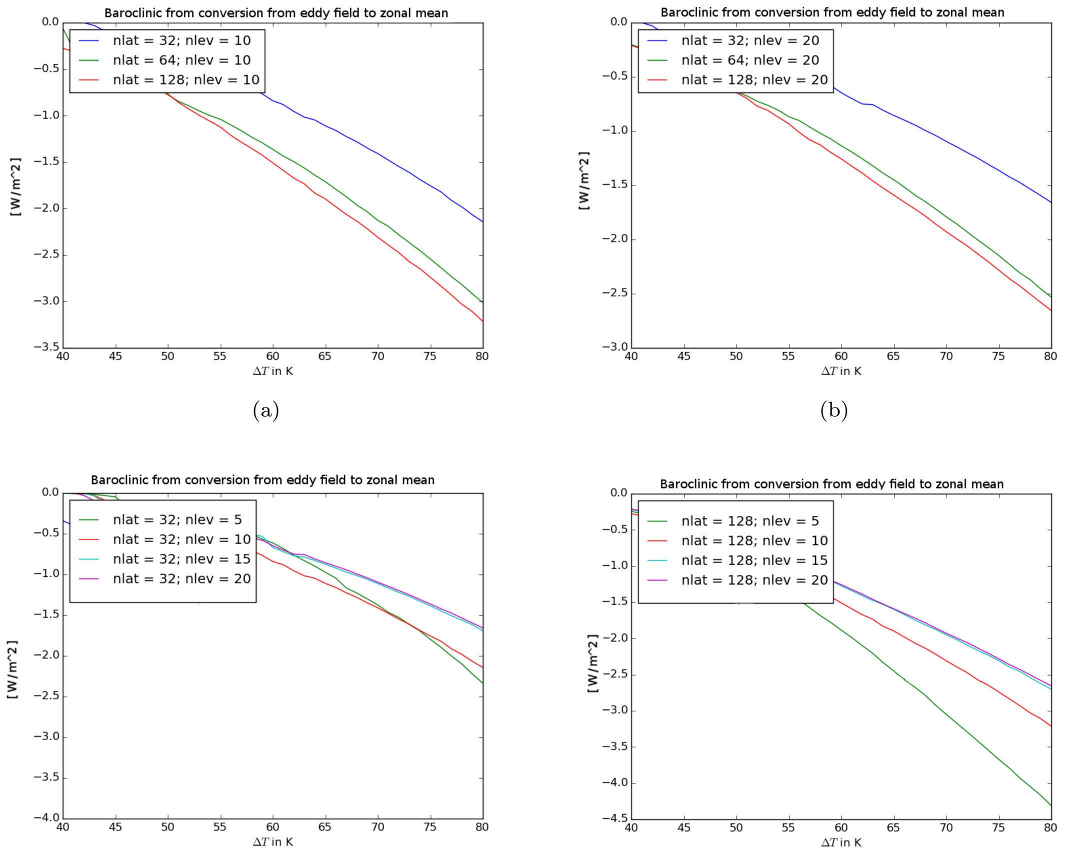


Figure 1: Panel (a) and panel (b) show how the leading LE changes when vertical resolution is fixed. Panel (c) and panel (d) show the LE as well but for fixed horizontal resolution. After a spinup time of 30 years these LEs are computed along a 30 year long trajectory

Figures 2 and 3 show the behavior of the baroclinic and barotropic conversions between the zonal mean and eddy field. The calculations are based on a program developed by Frank Lunkeit that uses the formulation of the LEC introduced in Ulbrich and Speth (1991). We observe an interesting difference how the conversions change depending on the resolution. Seemingly, both conversions converge for higher horizontal resolutions. Of course, these first preliminary results do not allow any further conclusions regarding the significance of this observation. Nevertheless, it suggests that the leading direction of linear instability (and maybe most of the fast growing directions) features very fast processes that are not directly linked to the classical growing Charney/Eady type of instabilities. This underlines how important it will be in future work to study and compare the spectral structure of the fast and slow growing CLVs in this model. We speculate that the slower growing linear instabilities (hence slower growing CLVs) which have lower Lyapunov exponents could feature a convergence behavior with respect to resolution that is closer to what is found for the baroclinic and barotropic conversions in the LEC, since there have been results suggesting these instabilities are qualitatively closer to the properties of the eddy field (see Schubert and Lucarini (2015)).



(a) (b) (c) (d)

Figure 2: Baroclinic Conversion from eddy field to zonal mean averaged vertically in W/m^2. Panel (a) and panel (b) show how the baroclinic conversion changes when the vertical resolution is fixed and the horizontal resolution varies. In panel (c) and panel (d) the horizontal resolution is fixed and the vertical resolution is varied.

Our limited time also allowed to study the spectrum of exponents in a little more detail. Based on the previous findings on resolution dependence, we conducted runs over a few years (model time) and computed the spectrum of the first 200 Lyapunov exponents. In view of the resolution dependence of the LEC and the LLE and being aware of some of the computational restrictions, we decided to compute the spectra for three distinct $\Delta T_{EP} = 40K$, $50K$, $70K$ and chose a vertical resolution of 15 levels and a horizontal resolution of 64 latitudes (128 longitudes, hence T42). Additionally, it was possible to study some of the basic properties of the spectrum, since the number of positive LEs was always well below 200 (see table 1). Note that, the Kaplan-Yorke Dimension is comparatively small considering that the full phase space dimension is approximately 90000. We can also see that the metric entropy increases the higher ΔT_{EP} is (also found in simpler quasi-geostrophic models (Lucarini et al., 2007; Schubert and Lucarini, 2015).

Table 1: Properties of the computed spectra

ΔT_{EP} [K]	Positive Exponents	Kaplan-Yorke Dimension	Metric Entropy [1/day]	$1/\lambda_1$ [day]
40	41	97.39	0.49	37.38
55	58	188.87	1.42	16.38
70	72	225.12	2.43	10.39

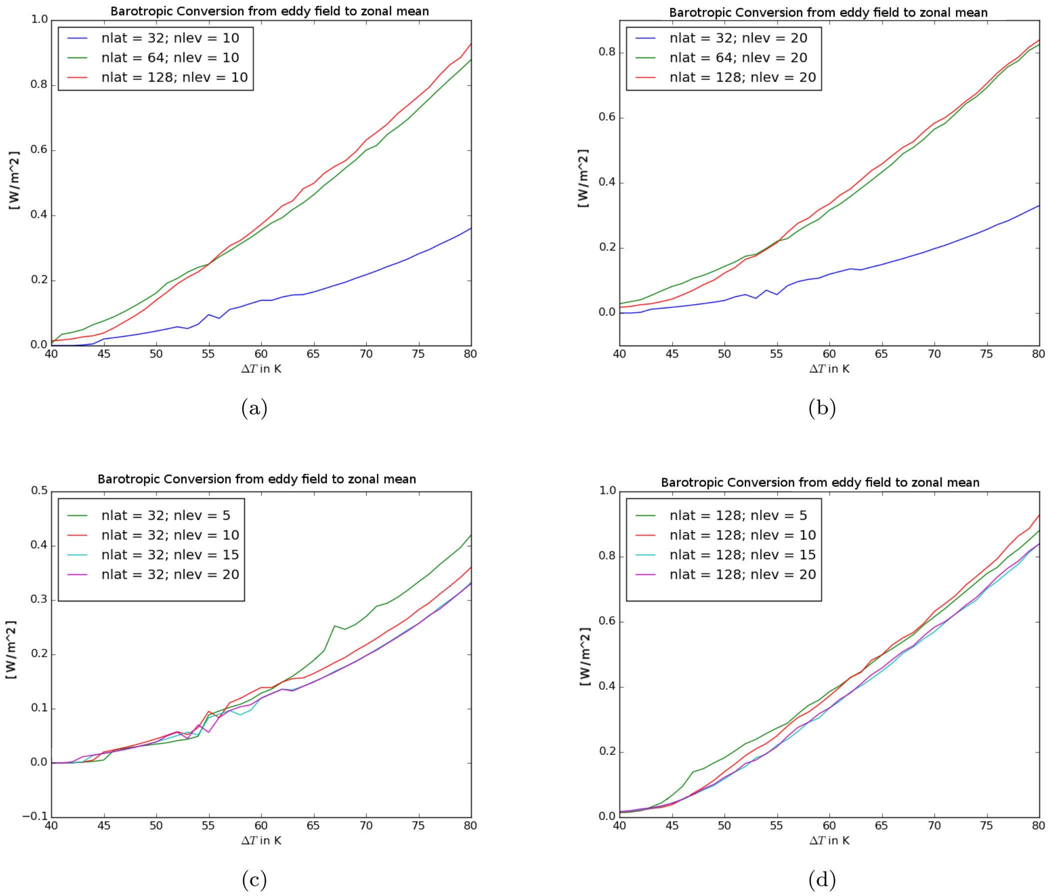


Figure 3: Barotropic Conversion from eddy field to zonal mean averaged vertically in W/m^2. Panel (a) and panel (b) show how the barotropic conversion changes when the vertical resolution is fixed and the horizontal resolution varies. In panel (c) and panel (d) the horizontal resolution is fixed and the vertical resolution is varied.

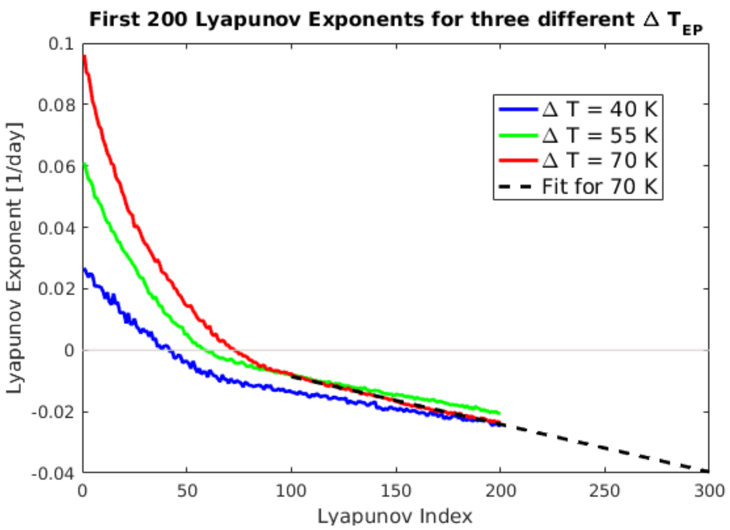


Figure 4: Results for the Lyapunov spectra. A linear fit for estimating the Kaplan-Yorke dimension was added for the red curve. This a choice based on the experience gathered in former experiments suggesting the existence of an almost linear slope in the LE spectrum in this region. Note that, since the dimension is 225.12 we only needed 26 exponents of the linear fit above 200.

References

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